IMPLEMENTATION, NUMERICS, AND APPLICATIONS OF DUALITY FOR OPTIMAL STOCHASTIC SWITCHING

PIYACHAT LEELASILAPASART

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CERTIFICATE OF ORIGINAL AUTHORSHIP

I, Piyachat Leelasilapasart, declare that this thesis, is submitted in fullfillment of the requirements for the award of the Doctor of Philosophy, in school of Mathematical and Physical Sciences at the University of Technology Sydney. This thesis is wholly my own work unless otherwise referenced or acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis. I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of the requirements for a degree at any other academic institution.

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Signature: Production Note:
Signature removed prior to publication.

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ABSTRACT

This thesis presents an implementation of algorithms using the scientific computing programing language, Julia. These algorithms are of the primal-dual kind and consequently gauge the distance of given approximate solutions to the optimal one.

The thesis explores a novel technique, based on a combination of primal and dual methods, to address complex problems. The primal (sub-gradient based approach) method for the approximate solution of discrete-time stochastic control problems is used to obtain approximate solutions. Furthermore, the duality method (pathwise dynamic programming) is combined to examine the numerical quality of solutions.

In the implementation, k-Means Clustering and Nearest Neighbor Search techniques are used to speed-up the computational effort. The use of the dynamic language programming Julia allows accurate procedures to find the optimal solution and returns tight confidence intervals with small standard errors.

The implementation package `rcss` is used to experiment with battery storage system problem and is published online via the author's GitHub. The topics of complex decision problems such as pairs trading, commodity rollover, and farm and agriculture management are discussed and developed for future research. Moreover, a variable reduction technique is considered, and concepts on its implementation have been developed in the thesis.

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